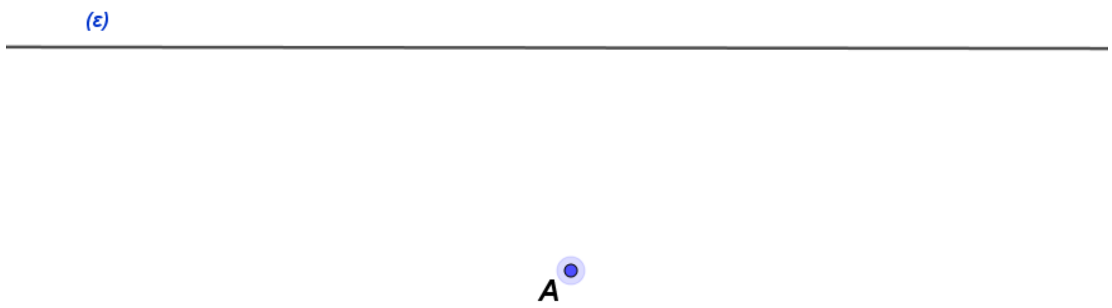
Laboratory Exercise

‘Frans van Schooten’s machine’’

Warming-up Phase

Find some points that have the same distance from the point A and the straight-line ε.

Can you assume where all these points with this property are situated?



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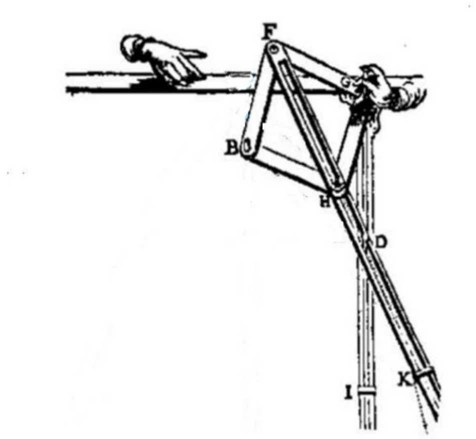
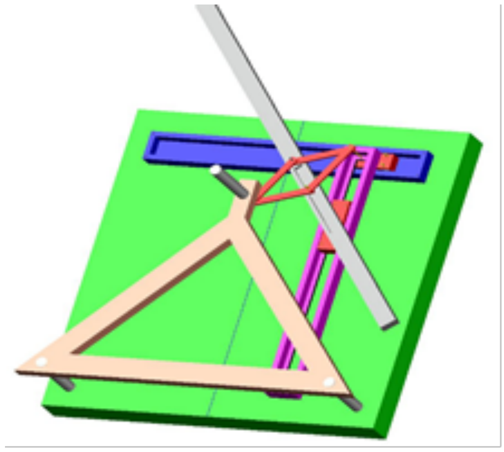
Phase A’

MATHEMATICAL MACHINES are special types of modular systems.

They are artefacts designed - usually by a mathematician - so that their articulated parts move and draw lines **following a mathematical law.**

By studying the mathematical machine and its movement, **we try to discover the ‘’hidden’’ law.**





The mathematical machine you see is a machine made by the mathematician Frans van Schooten (1615-1660).

Imagine a man holding the machine that is lying on a table. He is holding the machine with one hand, while his other hand - the hand you see on the G-joint – pushes towards left.

Pushing to the left, the G point drifts the vertical bar (GI).

The Point B is nailed to the table and fixed.

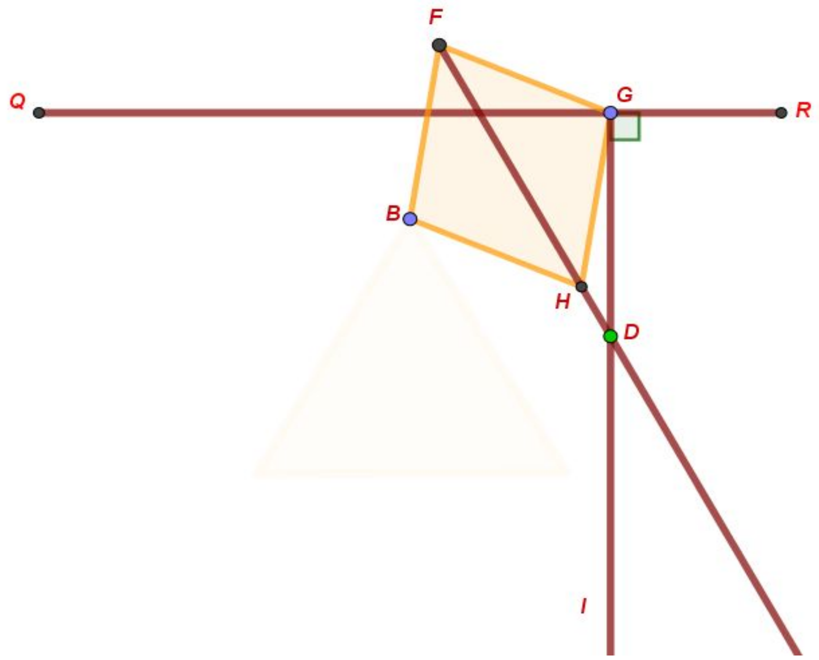
The horizontal bar is also nailed.

A pen that is placed at the point D, draws a line.

On the right side of the picture you can see a modern reconstruction of the **van Schooten’s machine.**

(**The large triangle was added to keep the construction stable. It does not play any role in the function of the machine, and we are not going to include it in our work**).

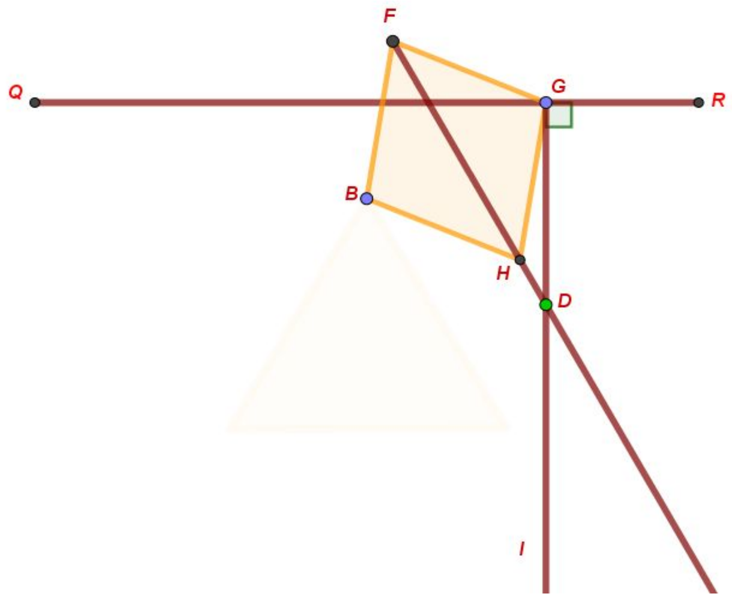
You see a geometric representation of the machine.



(1) What geometric shapes, and generally what geometric objects can you recognize in this representation?

(2) Write down and justify any relation that you can find.

(3) From the relations that arise, can you conclude something about the D point?



(4) The quadrilateral that exists in this system, what kind of quadrilateral is and why?

(5) Compare the triangles FBH and FGH. What are your conclusions?

(6) Compare the triangles BHD and GHD too. What are your conclusions?

(7) Which property of the point D is "unchanged" while the point G is moving?

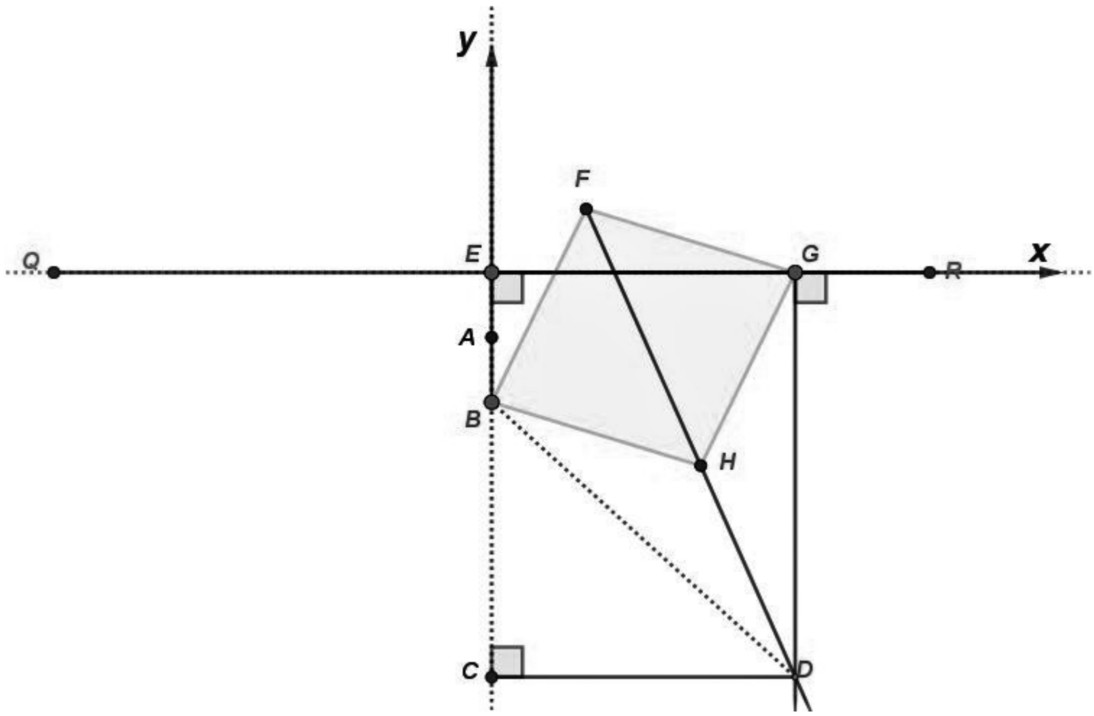
READ AGAIN THE FIRST QUESTION at the warm-up phase.

Can you now add more points to your answer? Justify your answer

Phase B'

In the figure below, we have placed Cartesian coordinate system axes

(1) Find the coordinates of the points **G & C**, if the fixed/immovable point **B** has coordinates **(0, -p)**, where **p = positive constant number,** and the point **D** has coordinates **(x, y).**



(2) Apply the Pythagorean theorem to the **BCD triangle** using the coordinates of points **B, C, D and G.**

(3) What relationship results (and links) for the coordinates of point D?

Formulate it and comment on it. Does it remind you of any well-known function?

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(4) Can you formulate a hypothesis on what kind of line would a pencil placed at point D draw? What is your reasoning for this hypothesis?

(5) What is the meaning of the constant term in the relationship you found?

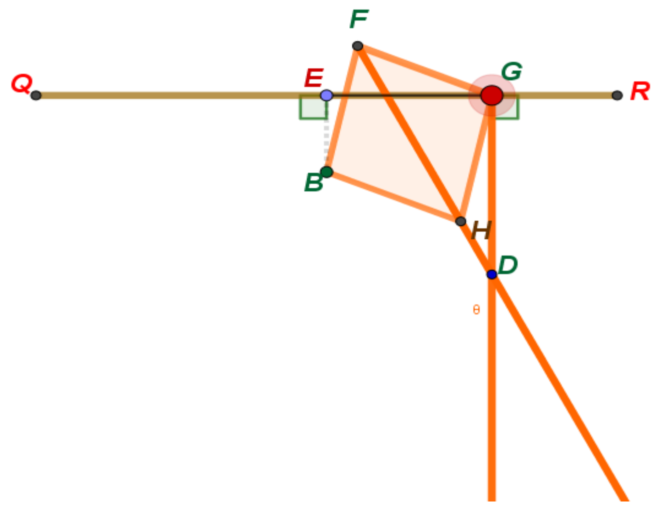
(6) What does the minus sign of the constant term mean?

(7) What does the negative coefficient of x^2 mean?

(8) What will happen to the coefficient of x^2 if the distance p is increased?

(9) What is the impact of this on the line that the machine is drawing?

VERIFICATION USING THE MACHINE



(1) Put the pen on point D and turn on the mathematical machine. What kind of line does it draw?

(2) Move the point B of the machine, **bringing it closer to point E.**

Then drag the point G along the QR bar. **What is the difference between the line being created now and the one you found before?**

Justify your answer using the relationship between the coordinates of the point D that you found in a previous query.

(3) Move the point B at a distance from the point E **larger than the initial distance**. Drag the point G along the QR bar. What is the difference among the line that is created now and the two previous ones?

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VERIFICATION USING GEOGEBRA

Verification Phase

Open the "Van Schooten.ggb" file located on the desktop of the PC that is a simulation of the Van Schooten machine.

Move the point G (holding the left mouse button firmly) to make the machine moving.

Laboratory Exercise

‘Frans van Schooten’s machine’’

ANSWERS

<u>Warming-up Phase</u>

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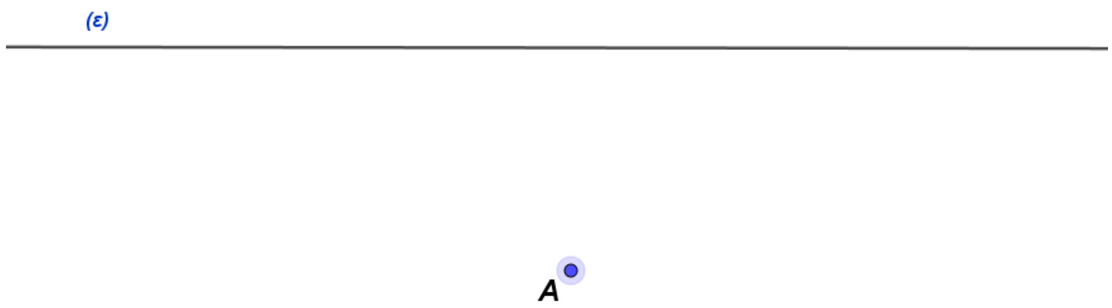

Phase A'

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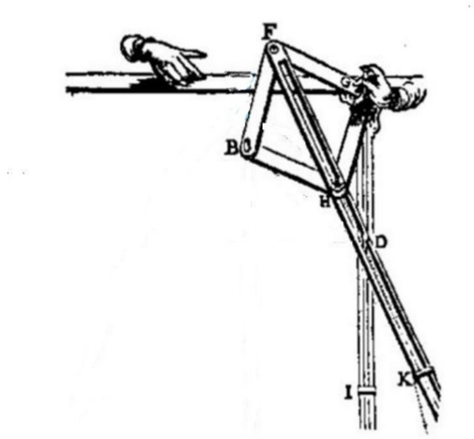

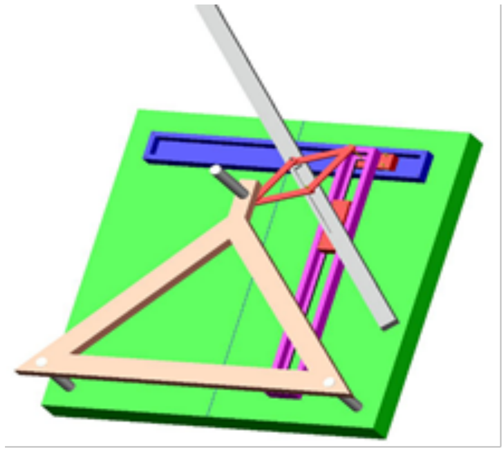

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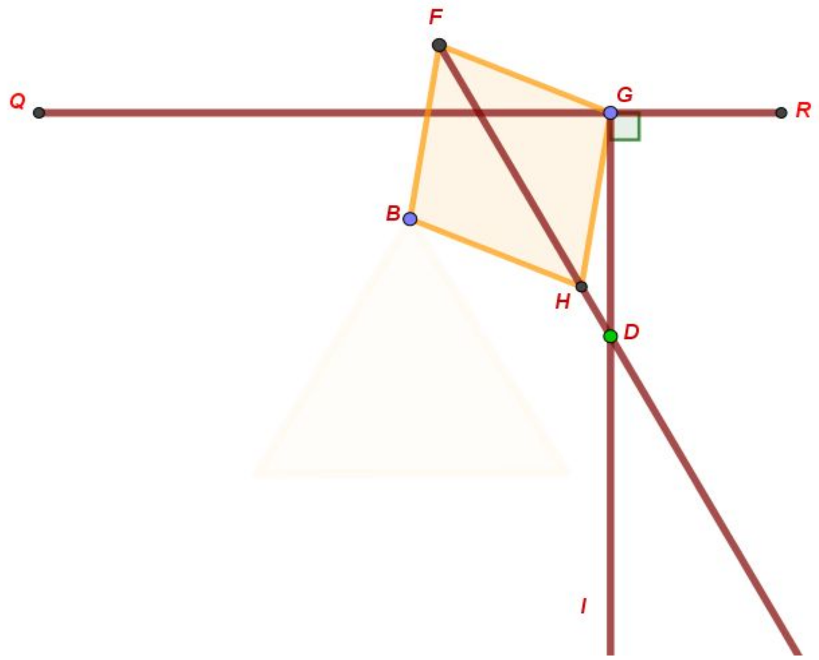


(1) What geometric shapes, and generally what geometric objects can you recognize in this representation?

ANSWER

(A) Quadrilateral FGHB, which resembles a Rhombus.

(B) Triangle GDH. (C) GI & FD semi-straight lines. (D) Right angle to G.

(E) Linear Segment QR.

(2) Write down and justify any relation that you can find.

(A) If they accept that the quadrilateral is Rhombus (by measurement) then the equality of the 2 triangles of Rhombus is obtained.

(B) If they draw the BD segment, then the equality of the GHD & BHD triangles is obtained, using the information given by the 2 Rhombus equality triangles.

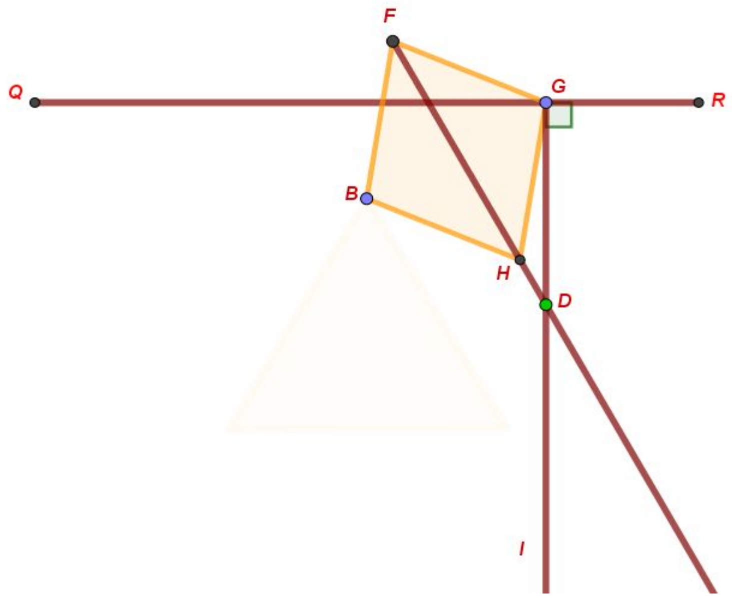
(3) From the relations that arise, can you conclude something about the D point?

ANSWER

(A) From the equality of the GHD & BHD triangles that came up in the previous question, the equality of the GD & BD segments can also be arised.

(B) is recalled perpendicularity of QR & GI.

(C) D therefore has equal distances from B and straight line QR (or segment).



(4) The quadrilateral that exists in this system, what kind of quadrilateral is and why?

ANSWER

(A) Quadrilateral FGHB, which looks like a Rhombus.

(5) Compare the triangles FBH and FGH. What are your conclusions?

ANSWER

(A) If they accept that the quadrilateral is Rhombus (by measurement) then the equality of the 2 triangles of Rhombus is obtained.

(6) Compare the triangles BHD and GHD too. What are your conclusions?

ANSWER

(B) If they draw the BD segment, then the equality of the GHD & BHD triangles is obtained, using the information given by the 2 Rhombus equality triangles.

(7) Which property of the point D is “unchanged” while the point G is moving?

ANSWER

(A) From the equality of the GHD & BHD triangles that came up in the previous question, the equality of the GD & BD segments can also be derived.

(B) is recalled perpendicularity of QR & GI.

(C) Therefore D is equidistant from B and the straight line (or segment) QR.

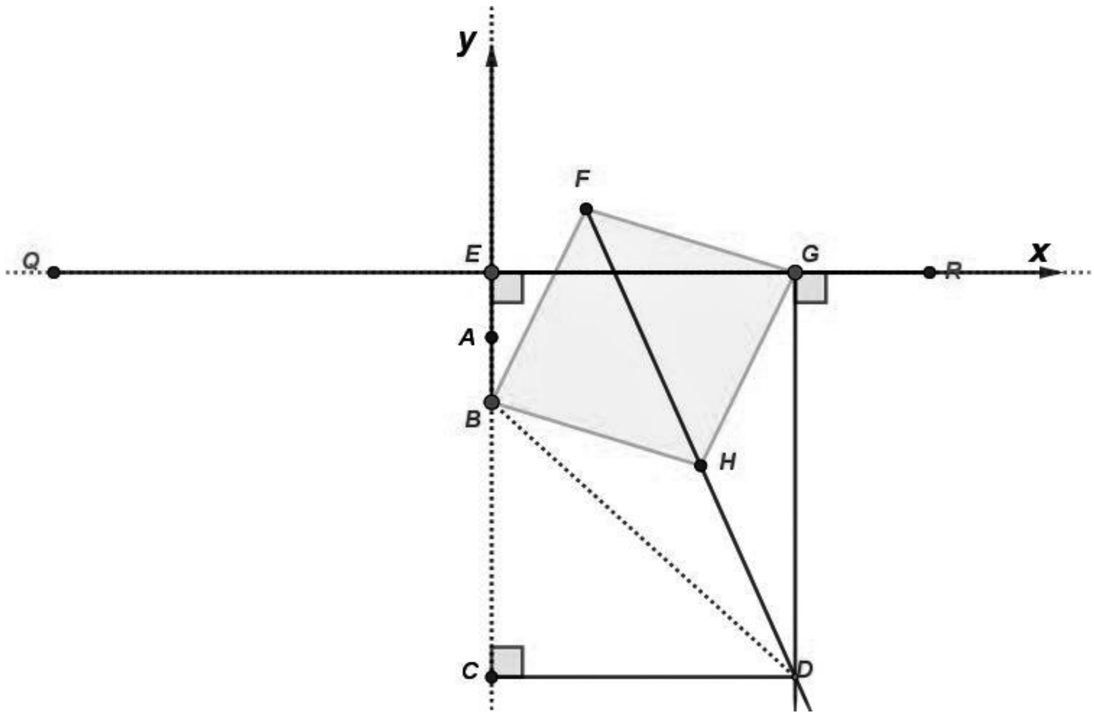
READ AGAIN THE FIRST QUESTION at the warm-up phase.

Can you now add more points to your answer? Justify your answer.

Phase B’

In the figure below, we have placed Cartesian coordinate system axes

(1) Find the coordinates of the points **G & C**, if the fixed/immovable point **B** has coordinates **(0, -p)**, where **p = positive constant number,** and the point **D** has coordinates **(x, y).**



(2) Apply the Pythagorean theorem to the **BCD triangle** using the coordinates of points **B, C, D and G.**

ANSWER

$$\mathbf{BD^2 = BC^2 + CD^2 \Rightarrow DG^2 = (CE - BE)^2 + CD^2 \Rightarrow DG^2 = (DG - BE)^2 + CD^2 \Rightarrow}$$

$$\mathbf{DG^2 = DG^2 - 2DG \cdot BE + BE^2 + CD^2 \Rightarrow BE^2 + CD^2 = 2DG \cdot BE}$$

$$\mathbf{\Rightarrow CD^2 = 2DG \cdot BE - BE^2 \Rightarrow CD^2 = 2DG \cdot p - p^2}$$

$$\mathbf{\Rightarrow |x|^2 = 2(-y) \cdot p - p^2}$$

(3) What relationship results (and links) for the coordinates of point D?

Formulate it and comment on it. Does it remind you of any well-known function?

ANSWER

$$|x|^2 = 2(-y)\cdot p - p^2 \Rightarrow x^2 = -2py - p^2$$
$$\Rightarrow 2py = -x^2 - p^2$$
$$\Rightarrow y = \frac{-1}{2p}x^2 - \frac{p}{2}$$

So we have the equation of the form y = $\alpha x^2 + \beta x + \gamma$ (parabola as "characterized" in the algebra of the 1st High School ....)

(4) Can you formulate a hypothesis on what kind of line would a pencil placed at point D draw? What is your reasoning for this hypothesis?

ANSWER

From the form of the equation connecting the coordinates (x, y) of point D, a parabolic curve will be drawn (as they give it in the 1st High School in Algebra's lesson).

(5) What is the meaning of the constant term in the relationship you found?

ANSWER

$y = \frac{-1}{2p}x^2 - \frac{p}{2}$. **Άρα** for x = 0 resulting $y = -\frac{p}{2}$.

At the point $(0, -\frac{p}{2})$ intersects the y'y axis, which is also the top of the curve.

(6) What does the minus sign of the constant term mean?

ANSWER

(A) The point A$(0, -\frac{p}{2})$ is on the negative axis of y'y.

(B) A is the middle of BE.

(Γ) A is at the highest point relative to the rest points of the curve.

(7) What does the negative coefficient of x^2 mean?

ANSWER

$$y = \frac{-1}{2p}x^2 - \frac{p}{2}$$

(A) for any value of x, a negative result for $y = \frac{-1}{2p}x^2$, which becomes more negative adding $-\frac{p}{2}$.

Therefore, the curve is ALL below x'x (QR bar).

(B) Are possible students to look at the monotony of the curve, working at 2 intersections.

In QE for negative x, which follows strictly increasing function $f(x) = \frac{-1}{2p}x^2 - \frac{p}{2}$.

In the ER for positive x, where the function decreases $f(x) = \frac{-1}{2p}x^2 - \frac{p}{2}$.

(8) What will happen to the coefficient of x^2 if the distance p is increased?

ANSWER

(A) $y = \frac{-1}{2p}x^2 - \frac{p}{2}$. So, if the distance p increases, then the coefficient of x^2 decreases in absolute value.

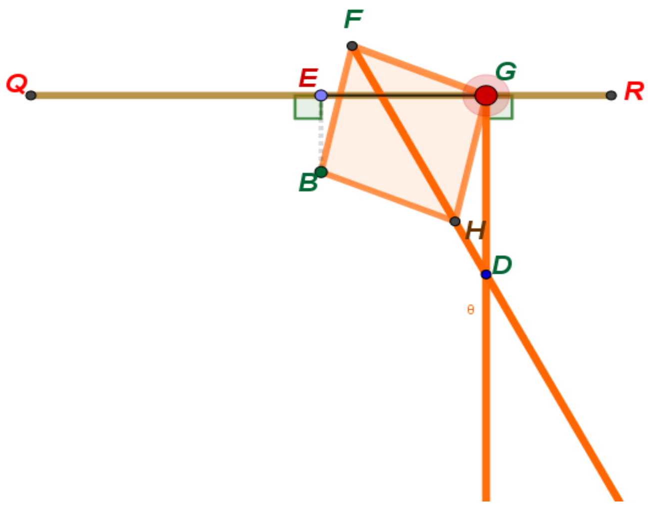
(9) What is the impact of this on the line that the machine is drawing?

ANSWER

(A) This can be interpreted as: 'for the same abscissa x of points G and D, we have lower ordinate in absolute value of point D, that is, a smaller distance of point D from x'x.

(B) That is, the curve “shallow” or “unburden”.

VERIFICATION USING THE MACHINE



(1) Put the pen on point D and turn on the mathematical machine. What kind of line does it draw?

ANSWER

Parabolic curve …… Parabola….

(2) Move the point B of the machine, **bringing it closer to point E**.

Then drag the point G along the QR bar. **What is the difference between the line being created now and the one you found before?**

Justify your answer using the relationship between the coordinates of the point D that you found in a previous query.

ANSWER

(A) $y = \frac{-1}{2p}x^2 - \frac{p}{2}$ **. So, if the distance p decreases, then the coefficient of x^2 increases.**

(B) This can be interpreted as: 'for the same abscissa x of point G and D, there is a bigger (absolute value) ordinate y of point D, that is, a greater distance of point D from x'x.

(C) that is "deeper" or "curved" curve.

(D) therefore, shows more "curved curve".

(3) Move the point B at a distance from the point E **larger than the initial distance**. Drag the point G along the QR bar. What is the difference among the line that is created now and the two previous ones?

Justify your answer using the relationship between the coordinates of the point D that you found in a previous query.

ANSWER

(A) $y = \frac{-1}{2p}x^2 - \frac{p}{2}$**. So, if the distance p increases, then the coefficient of** x^2 **decreases (in absolute value).**

(B) This can be interpreted that "the same abscissa x of the point G and D, shows a smaller absolute value in the ordinate y of the point D, i.e. a shorter distance of the point D from the x'x.

(C) That is, the curve "shallow".

VERIFICATION USING GEOGEBRA

Verification Phase

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